Area Formulas	Circumference
Transformations of $\sin(x)$, $\cos(x)$, $\csc(x)$, $\sec(x)$	Transformations of $\tan(x)$ and $\cot(x)$
Finding an Inverse $f^{-1}(x)$	Verifying Inverses
2 Properties of Inverses	Even/Odd Functions
Examples of Even Functions	Examples of Odd Functions

$$C = 2\pi r$$

$$C = \pi d$$

Circle: $A = \pi r^2$

Trapezoid: $A = \frac{h}{2}(b_1 + b_2)$

Triangle: $A = \frac{1}{2}bh$

Cylinder: $SA = 2\pi r^2 + 2\pi rh$

Sphere: $SA = 4\pi r^2$

$y = a\tan(bx) + c$

Amplitude = $|a|$

Period = $\frac{\pi}{b}$

Vertical Shift = c

*Same goes for cot(x)

$y = a\sin(bx) + c$

Amplitude = $|a|$

Period = $\frac{2\pi}{b}$

Vertical Shift = c

*Same goes for cos(x), sec(x), and csc(x)

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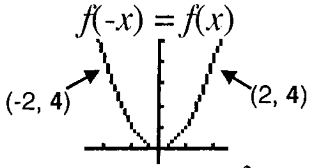
1. You can verify that two functions are inverses algebraically by seeing if their composition equals x.
   $f(f^{-1}(x)) = x$ or $f^{-1}(f(x)) = x$
2. You can verify that two functions are inverses graphically by seeing if they are symmetrical about the line $y = x$.

1. Interchange x and y in the equation
2. Solve for y. (This will not be possible if y cannot be written as a function of x)
3. Replace y with $f^{-1}(x)$

Even: Symmetrical about the y-axis

$f(-x) = f(x)$

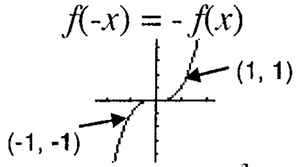


Example: $y = x^2$

y is the same whether you plug in x or $-x$

Odd: Symmetrical about the origin

$f(-x) = -f(x)$



Example: $y = x^3$

y is the opposite sign when you plug in x or $-x$

1. If the point $(\boldsymbol{a}, \boldsymbol{b})$ is on the graph of $f(x)$, the point $(\boldsymbol{b}, \boldsymbol{a})$ is on the graph of $f^{-1}(x)$.
2. The slopes of inverse functions are **reciprocals.**

Ex: Suppose $f(x)$ and $g(x)$ are inverses

1. If $f(2)=8$, then $g(8)=2$

If $f'(2) = 3/4$, then $g'(8) = 4/3$

$y = \sin x$

$y = \frac{x}{x^2 + 1}$

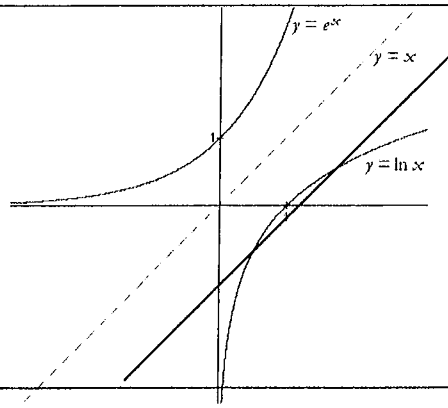
$y = x^5 - x^3$

$y = x^2$

$y = \cos x$

$y = \sqrt{9 - x^2}$ (Semi-Circle)

Finding Domain	Function Recognition $y = e^x$ and $y = \ln(x)$
Parallel Lines Perpendicular (Normal) Lines	Strategies for Finding a Limit
Horizontal Asymptote	Vertical Asymptote
A Function Is Continuous If…	Types of Discontinuities
Average Rate of Change	Instantaneous Rate of Change



Note: These functions are inverses.

Notice the symmetry about the line $y = x$

- You cannot divide by zero
- You cannot take the square root of a negative number
- You cannot take the ln or log of a negative number or zero.

Example: $y = \sqrt{9 - x^2}$ Domain: [-3, 3]

$y = \ln(x)$ Domain: $(0, \infty)$

1. Direct Substitution
2. Factor and Cancel
3. End Behavior Model (EBM). Use for $\lim\limits_{x \to \pm\infty} f(x)$
4. Rationalize the Numerator
5. L'Hôpital's Rule
6. Compare Left & Right Hand Limits (Piecewise Functions)

Parallel lines have the same slope

Perpendicular lines have negative reciprocal slopes

The line $x = a$ is a Vertical Asymptote of the graph of a function if either

$\lim\limits_{x \to a^-} f(x) = \pm\infty$ or $\lim\limits_{x \to a^+} f(x) = \pm\infty$.

Vertical Asymptotes exist at values of x that cause the function to be undefined.

Ex: $f(x) = \frac{1}{x}$ has a VA at $x = 0$

The line $y = b$ is a Horizontal Asymptote of the graph of a function if either

$\lim\limits_{x \to \infty} f(x) = b$ or $\lim\limits_{x \to -\infty} f(x) = b$.

Ex. $y = \frac{2x}{x+1}$ has a HA of $y = 2$.

1. Removable (Hole)
2. Jump
3. Infinite (Asymptote)
4. Oscillating

Note: If a function is differentiable at a given point it is also continuous at that point.
If a function is continuous at a given point it may or may not be differentiable at that point.

$$\lim_{x \to c} f(x) = f(c)$$

The limit of $f(x)$ as x approaches c equals the value at c, and they are finite.

Ex. $y = \begin{cases} x & x < 3 \\ -x + 6 & x \geq 3 \end{cases}$ is continuous at x=3

because $\lim\limits_{x \to 3^-} (x) = \lim\limits_{x \to 3^+} (-x + 6) = f(3)$

Derivative

Slope of a Tangent Line

On the Calculator:
[MATH] [8]

$$\text{Slope} = \frac{f(b) - f(a)}{b - a}$$

Slope of a Secant Line

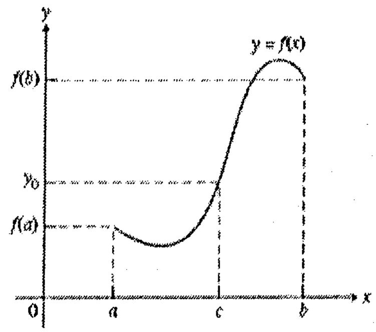
Intermediate Value Theorem	Mean Value Theorem
Piecewise Functions Continuous? Differentiable?	Position Velocity Acceleration
Particle Motion	Definition of a Derivative
Implicit Differentiation Tips	Derivative of e^x, a^x, $\ln\|x\|$, and $\log x$
Derivative of Inverse Trigonometric Functions	Trig Derivatives

If $f(x)$ is **continuous** on $[a, b]$ and **differentiable** at every point of its interior (a, b), then there is at least one point, c, in the interval which

$$\frac{f(b)-f(a)}{b-a}=f'(c)$$

Avg Rate of Change = Instantaneous Rate of Change

Slope of Secant Line = Slope of Tangent Line



A function that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$.

DERIVATIVE ↓

Position (units)

Velocity (units/time)

Acceleration (units/time^2)

↑ ANTI-DERIV

Example: $f(x)=\begin{cases} x^2-3x+9 & x\le 2 \\ kx+1 & x>2 \end{cases}$

Continuity: To find the value of k set both equations equal, plug in $x = 2$, and solve.
Differentiable: To find k take the derivative of each equation plug in $x = 2$, set them equal & solve.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \text{or} \quad \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

Ex: $\displaystyle\lim_{h\to 0}\frac{\tan 3(x+h)-\tan 3x}{h}$

Solution: $f(x) = \tan 3x$ so the derivative is $f'(x) = \mathbf{3\sec^2(3x)}$

Particle at Rest: Velocity = 0
Moving Forwards: Velocity is positive
Moving Backwards: Velocity is negative
Change Directions: Velocity changes sign (+ to − or − to +)
Speed Up: v<0 and a<0 OR v>0 and a>0
Slow Down: v and a signs differ

$$\frac{d}{dx}e^x=e^x \qquad \frac{d}{dx}a^x=a^x\ln a$$

$$\frac{d}{dx}\ln|x|=\frac{1}{x} \qquad \frac{d}{dx}\log_a x=\frac{1}{x\ln a}$$

Don't Forget Chain Rule!

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1. Remember to write y' when you take the derivative of y.
2. Be sure to notice product rule if it is there. If a product rule follows a negative sign USE PARENTHESIS.
3. Only solve for y' if asked to find dy/dx Otherwise sub in the point to find slope.

$$\frac{d}{dx}\sin x=\cos x \qquad \frac{d}{dx}\sec x=\sec x\tan x$$

$$\frac{d}{dx}\cos x=-\sin x \qquad \frac{d}{dx}\csc x=-\csc x\cot x$$

$$\frac{d}{dx}\tan x=\sec^2 x \qquad \frac{d}{dx}\cot x=-ccs^2 x$$

$$\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}x=-\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}x=-\frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x=\frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}x=-\frac{1}{|x|\sqrt{x^2-1}}$$

Basic Rules For Differentiation	Basic Rules For Differentiation (Continued)
Product and Quotient Rule For Differentiation	Derivative of $\ln[f(x)]$
First Derivative Test	Second Derivative Test
Second Derivative Test for Relative Extrema	Extreme Value Theorem
Volume Formulas (Often used for Related Rates Problems)	Relative Max Relative Min (Justify your answer)

4. $\frac{d}{dx}[cf(x)] = c\,f'(x)$

The derivative of a function times a constant multiple is the constant multiple times its derivative.

5. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

The derivative of a sum is the sum of the derivatives.

1. $\frac{d}{dx}(c) = 0$	The derivative of a constant is zero
2. $\frac{d}{dx}(x) = 1$	The derivative of x is 1
3. $\frac{d}{dx}(x^n) = nx^{n-1}$	Power Rule

$$\frac{d}{dx}\ln[f(x)] = \frac{f'(x)}{f(x)}$$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

The second derivative $f''(x)$ of a function can be used to determine where *f*(*x*) is:

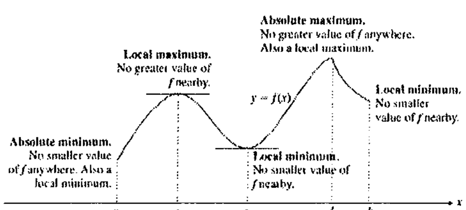
1. Concave Up ($f''(x)$ is positive)
2. Concave Down ($f''(x)$ is negative)
3. At an Inflection Point ($f''(x)$ changes sign, + to – or – to +)

FIRST determine where $f'(x)$=0 and where $f'(x)$DNE. State the domain & draw a sign line.

The original function, *f*(*x*), is:

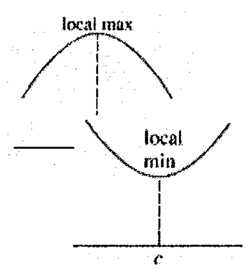
1. Increasing when $f'(x)$ is positive
2. Decreasing when $f'(x)$ is negative
3. At a maximum when $f'(x)$ goes from + to -

If a function is **continuous** on a **closed interval** [*a*, *b*] then it has a maximum and a minimum value on the interval.



This is an alternate $f''(x)$ way to determine the max/min of a function.

1. If $f'(c) = 0$, and $f''(c) < 0$, *f* has a local max at $x = c$.
2. If $f'(c) = 0$, and $f''(c) > 0$, *f* has a local min at $x = c$.



f(*x*) has a relative max at x = __ because $f'(x)$ goes from + to – at x = __.

f(*x*) has a relative min at x = __ because $f'(x)$ goes from – to + at x = __.

Cube: $V = x^3$

Cylinder: $V = \pi r^2 h$

Cone: $V = (1/3)\,\pi r^2 h$

Sphere: $V = (4/3)\,\pi r^3$

Rectangular Prism: $V = LWH$

Newton's Method	RAM
Trapezoidal Rule (Uneven Intervals)	Trapezoidal Rule (Even Intervals)
Average Value of a Function	Integration Techniques
Integration of Basic Trigonometric Functions	Integration of Basic Trigonometric Functions (Continued)
Integration of $\tan(u)$ and $\cot(u)$	Integration of $\frac{1}{u}$, e^u, a^u

Rectangular Approximation Method (Estimates Area Under a Curve)

Time(sec)	10	15	30	60	90
Rate(m/s)	15	20	25	30	35

LRAM=(5)(**15**)+(15)(**20**)+(30)(**25**)+(30)(**30**)

RRAM = (5)(**20**)+(15)(**25**)+(30)(**30**)+(30)(**35**)

MRAM = (30-10)(**20**) + (90-30)(**30**)

Used for determining the zeros of a function.

1. Call the original function $f(x)$. Calculate the derivative $f'(x)$
2. Evaluate $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ using the initial approximation to find x_2.
3. Repeat!

$$T = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right)$$

Time(sec)	10	15	20	25	30
Rate(m/s)	15	20	25	30	35

T = (5/2) [15 + 2(20) + 2(25) + 2(30) + 35] T = 500 meters

Time(sec)	10	15	30	60	90
Rate(m/s)	15	20	25	30	35

T = ½(15+20)(5) + ½(20+25)(15)…
(uneven intervals...use individual trapezoids)

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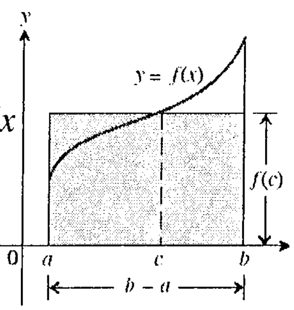
1. Anti-derivatives
2. U-Substitution

On the Calculator:
[MATH] [9]

$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$



Units may be (for example):

ft / sec   Tickets sold/ hr

m/sec   Words typed/min

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \tan u \, du = \ln|\sec u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

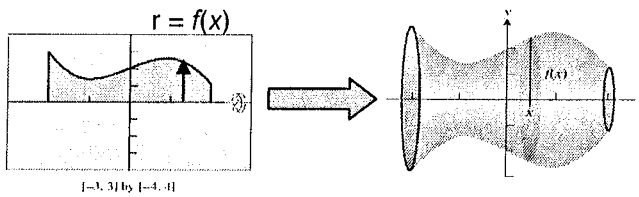
Integration of Logarithmic Forms	Integration Power Rule
Fundamental Theorem of Calculus Part I (Example Problem)	Volumes of Revolution (Around the x-axis) Disks!
Volume by Cross Sections	Volumes of Revolution (Around the x-axis) Washers!
Interpretation of $\int_0^b \lvert R(t) \rvert \, dt$	Interpretation of $\int_0^b R(t) \, dt$
Interpretation of $\frac{1}{b-a} \int_a^b f'(x) dx$	L'Hôpital's Rule

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, \qquad n \neq 1$$

$$\int \ln u \, du = u \ln u - u + C$$

$$\int u^n \ln u \; du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$$

$$\int \frac{1}{u \ln u} du = \ln | \ln u | + C$$



$$V = \pi \int_a^b r^2 dx$$

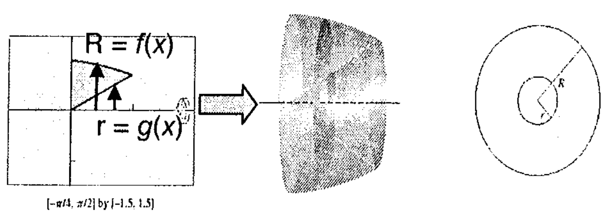
If f is the function given by

$$f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt \text{, then } f'(2) =$$

You are asked to evaluate the derivative of an anti-derivative!

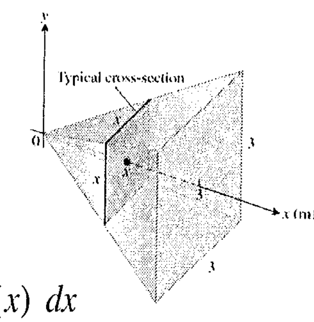
$$f'(x) = \sqrt{(2x)^2 - 2x} \bullet 2$$

$$f'(2) = \sqrt{(4)^2 - 4} \bullet 2 \;=\; 2\sqrt{12}$$



$$V = \pi \int_a^b R^2 - r^2 dx$$

1. Find a formula for $A(x)$, a typical cross section
2. Find the limits of integration
3. Integrate $A(x)$ to find the volume



$$V = \int_a^b A(x)\; dx$$

Suppose $R(t)$ is the rate, in miles per minute, that a student rides a bike.

$$\int_0^b R(t)\, dt$$

Interpretation: This is the net distance (in miles) the student travels during the first b minutes.

Suppose $R(t)$ is the rate, in miles per minute, that a student rides a bike.

$$\int_0^b |R(t)|\, dt$$

Interpretation: This is the total distance (in miles) the student travels during the first b minutes.

Suppose that

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

Suppose $f'(x)$ is the rate at which water flows into a tank

$$\frac{1}{b-a} \int_a^b f'(x) dx$$

Is the average value of the rate at which the water enters the tank (context) in gallons/min (units) over the interval $[a, b]$

Fundamental Theorem of Calculus – Part I	Fundamental Theorem of Calculus – Part II
General Power Rule & Constant Multiple Rule For Differentiation	Derivatives of Inverses
U-Substitution	Integration of sec(u) and csc(u)
Integration of Inverse Trig Forms	Integration of Exponential Forms
Integration of Exponential/ Trigonometric Forms	Integration by Parts Formula

Assume that $f(x)$ is continuous on $[a, b]$ and let $F(x)$ be an antiderivative of $f(x)$ on $[a, b]$. Then $\int_a^b f(x)dx = F(b) - F(a)$	Assume that $f(x)$ is a continuous function on $[a, b]$ Then the area function $A(x) = \int_a^x f(t)dt$ is an antiderivative of f(x), that is $A'(x) = f(x)$ or equivalently $\frac{d}{dx}\int_a^x f(t)dt = f(x)$. Furthermore, $A(x)$ satisfies the initial condition $A(a) = 0$
$g'(x) = \frac{1}{f'(g(x))}$ Where $g(x)$ is the inverse of $f^{-1}(x)$	General Power Rule: $\frac{d}{dx} f(x)^n = nf(x)^{n-1} f'(x)$ Constant Multiple Rule: $\frac{d}{dx} f(kx+b) = kf'(kx+b)$
$\int \sec u\, du = \ln \lvert \sec u + \tan u \rvert + C$ $\int \csc u\, du = \ln \lvert \csc u - \cot u \rvert + C$	If an integral has the form $f(u(x))u'(x)$, then rewrite the entire integral in terms of u and its differential $du = u'(x)\,dx$: $\int f(u(x))u'(x)\,dx = \int f(u)\,du$
$\int ue^{au} du = \frac{1}{a^2}(au-1)e^{au} + C$ $\int u^n e^{au} du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au} du$	$\int \frac{du}{\sqrt{a^2 - u^2}} du = \sin^{-1}\frac{u}{a} + C$ $\int \frac{du}{a^2 + u^2} du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$
$\int u(x)v'(x)dx =$ $u(x)v(x) - \int u'(x)v(x)dx$	$\int e^{au} \sin bu\, du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$ $\int e^{au} \cos bu\, du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$